ELSEVIER

International Journal of Heat and Fluid Flow 19 (1998) 644–654

International Journal of HEAT AND FLUID FLOW

Optimization of heat transfer through finned dissipators cooled by laminar flow

Giampietro Fabbri

Dipartimento di Ingegneria Energetica, Nucleare e del Controllo Ambientale, Università degli Studi di Bologna, Via Zannoni 45/2, 40134 Bologna, Italy

Received 7 August 1997; accepted 17 May 1998

Abstract

In the present work, the problem of optimizing the shape and the spacing of the fins of a thermal dissipator cooled by a fluid in laminar flow is studied. For a particular finned conduit, the velocity and temperature distributions on the transversal section are determined with the help of a finite element model and a global heat transfer coefficient is calculated. A polynomial lateral profile is proposed for the fins and the geometry is optimized in order to make the heat transfer coefficient as high as possible with the smallest dimensions or the lowest hydraulic resistance to the flow. The optimum fin profile and spacing, obtained by means of a genetic algorithm, are finally shown for different situations. Increases of 45% are obtained in the heat transfer coefficient referring to the maximum values which can be obtained with rectangular fin profiles.

Notation

a	fin height (m)
b	fin base thickness (m)
c_p	coolant specific heat (J/kg K)
d	distance between the fin base and the opposite flat wall (m)
e	width of the portion of conduit section (m)
E_c	compared effectiveness
f	half-width of the fins (m)
F'_v	viscous force per unit of length (N/m)
m_{Aik}	elements of the surface averaging matrix (m^2)
m_{Mik}	elements of the momentum transportation matrix
m_{Hik}	elements of the heat transportation matrix
h	global heat transfer coefficient (W/m^2 K)
h_r	global heat transfer coefficient of a reference flat wall conduit (W/m^2 K)
k_c	thermal conductivity of the coolant (W/m K)
k_f	thermal conductivity of the finned plate (W/m K)
l_i	perimeter crossed by q'' around the ith node (m)
n	polynomial order
Nu_e	equivalent Nusselt number
p	generalized pressure (N/m^2)
q''	heat flux per unit of surface uniformly imposed on the flat side of the finned plate (W/m^2)
q'_{k_i}	conductive heat flux which enters the ith node per unit of length (W/m)
q'_{c_i}	convective heat flux which leaves the ith node per unit of length (W/m)
R'_{p_i}	Resultant of the pressure forces acting on the ith node per unit of length (N/m)
R'_{v_i}	Resultant of the viscous forces acting on the ith node per unit of length (N/m)
t_i	temperature of the ith node (K)
T_b	bulk temperature of the coolant (K)
T_c	temperature of the coolant (K)
T_f	temperature of the finned plate (K)
T_{max}	maximum temperature of the cooled surface (K)
u	coolant velocity (m/s)
u_i	coolant velocity of the ith node (m/s)
w_i	coolant volume flow rate of the ith node (m^3/s)
w_t	total coolant volume flow rate (m^3/s)
x	longitudinal coordinate (m)
y	coordinate parallel to fin height (m)
y_i	coordinate of the ith node in the y direction (m)
z	coordinate orthogonal to fin height (m)
z_i	coordinate of the ith node in the z direction (m)

Greek

α	normalized height of the fins, a/d
β	normalized thickness of the finned plate base, b/d
γ	ratio of finned plate to coolant thermal conductivity, k_f/k_c
ϵ	normalized width of the portion of conduit section, e/d
ζ	normalized hydraulic resistance, defined by Eq. (30)
η	normalized coordinate parallel to fin height, y/d
μ	dynamic viscosity (Pa s)
ρ	coolant density (kg/m^3)
$\overline{\sigma}$	finned plate normalized average thickness, defined by Eq. (27)
ϕ	normalized half-width of the fins, f/d
$\overline{\phi}$	normalized average half-width of the fins, defined by Eq. (26)
ϕ_i	fin profile describing parameters
ψ_i	polynomial coefficients

0142-727X/98/$ – see front matter
PII: S 0 1 4 2 - 7 2 7 X (9 8) 1 0 0 3 6 - X

1. Introduction

Finned dissipators are commonly used in many engineering sectors, where high heat fluxes must be transferred. In new applications, such as in the electronic industry (Bar-Cohen and Kraus, 1990) or in the compact heat exchanger field (Kays and London, 1984), the necessity of reducing the volume of thermal dissipators has become even more important, in order to remove high heat fluxes from very small components. Much research has been carried out to increase the heat transfer effectiveness of heat dissipators, but the problem of optimizing the profile and the spacing of the fins has not yet been completely solved.

A criterion for optimum longitudinal fin profile was proposed by Schmidt (1926), who suggested the adoption of a parabolic shape. Such a criterion was confirmed by Duffin (1959) on the basis of a rigorous variational method. Afterwards, many authors contested Schmidt's conclusion (Maday, 1974; Snider and Kraus, 1987), which was correct from the point of view of the utilized model, but scarcely corresponding to the real phenomenon characteristics. Since then many fin profiles have been proposed, mainly parabolic or triangular, but without giving a final solution to the optimization problem and leaving doubts regarding the structural integrity of heat removers with an excessively decreasing profile (Tsukamoto and Seguchi, 1984). Recently, undulated fin profiles have been proposed (Snider et al., 1990; Spiga and Fabbri, 1994; Fabbri and Lorenzini, 1995) and a parabolic-undulated fin has been demonstrated as having a noticeably improved effectiveness.

Most of the studies performed on the optimization of the longitudinal fin profile consider a constant (Fabbri, 1997) or a temperature variable (Chung and Iyer, 1993; Yeh, 1994) heat transfer coefficient between the fin surface and the coolant fluid, and neglect the variations of this coefficient which are fluid dynamically related with the fin shape and spacing. When finned dissipators are located in channels where a coolant fluid passes through in laminar flow, such alterations are, on the contrary, not negligible. In this work, we then study the problem of optimizing heat transfer in a conduit composed of a finned plate opposite to a flat insulated wall with a coolant in laminar flow. Optimum fin spacing and lateral profile are found by using a finite element model and a genetic algorithm and taking into account variations in the local heat transfer coefficient induced by fluid flow.

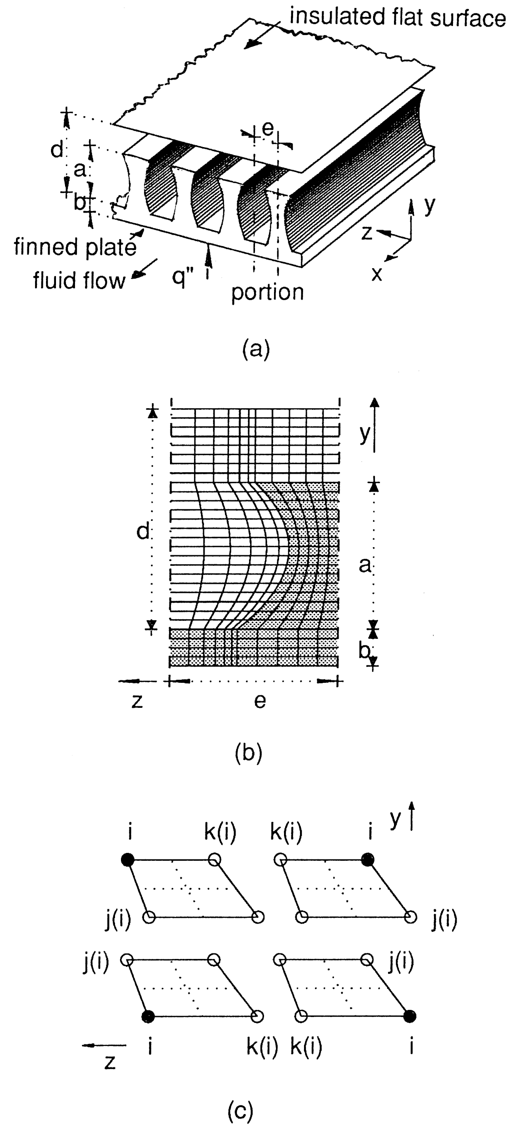

Fig. 1. Geometry of the heat removing system: (a) view of the finned conduit, (b) portion of the transversal section subdivided in finite elements, and (c) subdivision of an element in four sub-elements and indices for Eqs. (4) and (5).

2. The finned conduit model

Let us consider a conduit composed of a finned plate and a flat surface, which are infinitely long and wide (Fig. 1(a)). The flat surface is thermally insulated, while heat flux q'' flows through the finned plate, being uniformly imposed on the flat side opposite to the fins. A coolant in laminar flow passes through the conduit parallel to the fins. All fins are identical and have an axial symmetrical section in the plane which is orthogonal to the fluid flow direction.

We can determine the heat transfer performance of the system by studying a portion of it delimited by two symmetry axes (Fig. 1(b)). Let us choose an orthogonal coordinate system with the x axis parallel to the coolant flow and the y axis orthogonal to the conduit surfaces. As shown in Fig. 1(a), let a be the fin height and $f(y)$ an arbitrary function which describes the lateral fin profile. Moreover, let b be the fin base thickness, e the distance between the symmetry axes and d the distance between the fin base and the flat surface.

Let us introduce the following idealizations to formulate the problem:

- the system is in steady state,
- velocity and temperature profiles are completely developed,
- fluid and solid properties are uniform,
- viscous dissipation within the fluid is negligible,
- natural convection is negligible in regard to the forced convection.

Under such conditions, the velocity vector u is parallel to the x axis and is constant in the x direction. The coolant flow is then described by the following equation:

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}, \qquad (1)$$

where μ is the dynamic viscosity and p the generalized pressure, which includes the gravitation potential. Eq. (1) must be integrated by imposing boundary condition as follows: the velocity is zero on the contact surface between the fluid and the finned plate, and the partial derivative of the velocity in the normal direction is zero on the symmetry axes.

Since the heat flux q'' is uniform and the thermal profile is fully developed, the temperatures of the fluid and of the solid linearly change with the x coordinate. The conductive heat flux in the x direction is constant and can be neglected in an overall energy balance. Therefore, the temperature distribution in the coolant is described by the following equation:

$$\frac{\partial^2 T_c}{\partial y^2} + \frac{\partial^2 T_c}{\partial z^2} = \frac{\rho c_p}{k_c} u \frac{\partial T_c}{\partial x}, \tag{2}$$

where ρ is the density, c_p the specific heat and k_c the thermal conductivity of the coolant.

The temperature distribution in the finned plate is instead described by Laplace equation:

$$\frac{\partial^2 T_f}{\partial y^2} + \frac{\partial^2 T_f}{\partial z^2} = 0. \tag{3}$$

Eqs. (2) and (3) must be integrated by imposing the following boundary conditions: T_c is identical to T_f on the contact surface between the coolant and the finned plate, and the heat flux in the normal (to the surface) direction in the fluid is identical to that in the solid on the same surface. Moreover, the heat flux in the normal direction must be zero on the symmetry axes and on the insulated wall; while it must be equal to q'' on the flat surface of the finned plate. Finally, the value of the temperature in one point of the section is needed.

The problem can be solved numerically, resorting for example to a finite element method such as the following one. Let us subdivide the conduit portion in an array of trapezoidal elements by locating some nodes. An approximate example of subdivision is shown in Fig. 1(b). A large grid has been drawn in the figure for better comprehension. The elements of the coolant which are near the longitudinal fin surface can be more closely spaced in the z direction, since higher changes are expected in the velocity and temperature in the boundary region. In the y direction, the elements should always be closely spaced in order to follow the fin profile without excessive distortions.

In each element of the coolant, the velocity vector can be approximated by an interpolation of the values u_i, which it assumes in the four nodes of the element (Fig. 1(c)):

$$u(y,z) = \sum_i \frac{y - y_{j(i)}}{y_i - y_{j(i)}} \frac{z - z_{k(i)}}{z_i - z_{k(i)}} u_i, \tag{4}$$

where y_i, y_j, z_i, z_k are node coordinates. The element can be subdivided into four sub-elements by joining the middle points of the opposite sides (as shown in Fig. 1(c)). On the subdividing lines, the viscous force acting on the sub-elements per unit of length of the conduit can be calculated in the following way:

$$F'_v = \mu \sum_i \frac{\left[\int (z - z_{k(i)}) \mathrm{d}z - \int (y - y_{j(i)}) \mathrm{d}y\right]}{(y_i - y_{j(i)})(z_i - z_{k(i)})} u_i. \tag{5}$$

The resultant of all the viscous forces, which act on the sub-elements around a node can be written as

$$R'_{v_i} = \mu \sum_k m_{Mik} u_k, \tag{6}$$

the k index being extended to the node i and to those which surround it. The parameters m_{Mik} depend on the coordinate of these nodes and of those which delimit the sub-elements around the node i. For the nodes on the symmetry axes, m_{Mik} must be calculated by taking the boundary conditions into account. Since the velocity profile is fully developed, the resultant of the viscous forces must be balanced by the resultant of the pressure forces:

$$R'_{p_i} = S_i \frac{\mathrm{d}p}{\mathrm{d}x}, \tag{7}$$

S_i being the total transversal surface of all the sub-elements around the node i. Letting R'_{v_i} be opposite to R'_{p_i} for all the nodes, the following system is obtained:

$$M_M * U = \frac{1}{\mu} \frac{\mathrm{d}p}{\mathrm{d}x} A, \tag{8}$$

having gathered in matrix M_M the elements m_{Mik}, in vector U the velocities u_i and in vector A the surfaces S_i. Vector U can now be partitioned by grouping all the known terms in the subvector U_1 and the unknowns in U_2. Matrix M_M and vector A must be consequently partitioned. From Eq. (8), the following one is then obtained:

$$M_{M22} * U_2 = \frac{1}{\mu} \frac{\mathrm{d}p}{\mathrm{d}x} A_2 - M_{M21} * U_1. \tag{9}$$

The velocity distribution in the portion of the conduit section is determined after solving the system of Eq. (9).

In each element, the temperature can be approximated by an interpolation of the temperature t_k of the nodes with the same form factors of Eq. (4). Due to the analogy between the momentum transfer and heat transfer, the conductive heat flux which enters the sub-elements around a node for a unitary conduit length results as following:

$$q'_{k_i} = k_c \sum_k m_{Hik} t_k + l_i q'', \tag{10}$$

l_i being the total perimeter of the subelements which is crossed by q''. For the internal nodes of the coolant, parameters m_{Hik} are identical to m_{Mik}. For the node of the finned plate, parameters m_{Hik} must be calculated taking the ratio γ between the thermal conductivity of the solid and that of the fluid into account. Moreover, the convective heat flux in the longitudinal direction is

$$q'_{c_i} = \rho w_i c_p \frac{\partial T_c}{\partial x}, \tag{11}$$

w_i being the volume flow rate through the subelements around the node i. The volume flow rate is zero for all internal nodes of the finned plate. For other nodes, it can be calculated by integrating Eq. (4) on the surface of the sub-elements around each node. Then results

$$w_i = \sum_k m_{Aik} u_k. \tag{12}$$

The partial derivative in the x direction of the temperature is constant on the transversal section and can be written as a function of q'':

$$\frac{\partial T_c}{\partial x} = \frac{q'' e}{\rho w_t c_p}, \tag{13}$$

w_t being the total volume flow rate through the conduit section. The convective heat flux then results as

$$q_{c_i} = \frac{q'' e}{w_t} \sum_k m_{Aik} u_k. \tag{14}$$

In steady state, the conductive heat flux which enters the sub-elements in the transverse direction must be balanced by the convective heat flux in the longitudinal direction. By letting q'_{k_i} be equal to q'_{c_i} for all the nodes, the following system is obtained:

$$M_H * T = N, \tag{15}$$

$$N = \frac{q''}{k_c} \left(\frac{e}{w_t} M_A * U - L \right) \tag{16}$$

having gathered parameters m_{Hik} and m_{Aik} in matrices M_H and M_A, respectively, the temperatures t_k of all the nodes in vector T and the perimeters l_k in vector L. Vector U has now been extended to include the finned plate internal node velocities, which are zero. Vector T can be partitioned by putting the temperature of an arbitrary node (for example that with coordinates ($-b$,e), in which the maximum of the temperature is expected) in the subvector T_1 and the other temperatures in the subvector T_2. Matrix M_T and vector L must be consequently partitioned. From Eq. (16), the following one is then obtained:

$$M_{H22} * T_2 = N_2 - M_{H21} * T_1. \tag{17}$$

The temperature distribution in the portion of the conduit section is determined as a function of T_1 after solution of system of Eq. (17).

Letting the product of velocity and temperature be approximated in each element by an interpolation of the values which it assumes in the nodes with the same form factors of Eq. (4), the bulk temperature can be calculated as follows:

$$T_b = \frac{1}{w_t} \sum_i \sum_k m_{Aik} u_k t_k \tag{18}$$

the index i being extended to all the nodes of the coolant.

Due to the linearity of the system, an arbitrary value can be assigned to T_1 in order to calculate the global heat transfer coefficient of the finned conduit.

3. Dimensioning criteria

Most of the thermal power dissipation problems consist in extracting as high a heat flux as possible from a surface by keeping its temperature under a required value. The performance of heat dissipators must then be evaluated paying particular attention to the maximum drop between the temperature of the surface which is cooled and the bulk temperature of the coolant.

For the system described in the previous section, an appropriate global heat transfer coefficient can be defined as follows:

$$h = \frac{q''}{T_{max} - T_b} \tag{19}$$

T_{max} being the maximum temperature on the surface which is cooled, i.e., the surface on which q'' is imposed. Temperatures T_{max} and T_b must be calculated at the same value of the x coordinate. Moreover, it is possible to define an equivalent Nusselt number:

$$Nu_e = \frac{h\,2\,d}{k_c} \tag{20}$$

which corresponds to the Nusselt number which would be calculated if the same heat flux q'' were dissipated through a flat surface with a zero thickness at the distance d from the insulated surface. Such an equivalent Nusselt number is a function of the ratio γ for the finned conduit. A criterion for optimizing the dissipator dimensions and shape can then consist in maximizing Nu_e. In this way, at a given coordinate x, for a given bulk temperature and an established distance d, the finned conduit dissipates the maximum heat flux without exceeding the maximum temperature limit on the surface being cooled.

In order to respect the temperature limit for an extended length, moreover, it is expedient to let the bulk temperature hardly increase with the x coordinate, that is obtained by increasing the mass flow rate as much as possible. If the pressure drop between the beginning and the end of the conduit is constrained, the maximum mass flow rate is obtained in correspondence with the lowest hydraulic resistance. It is then useful to dimension the dissipator aiming to increase Nu_e and at the same time to keep the hydraulic resistance as low as possible.

Therefore, when the pressure drop between the beginning and the end of the conduit is constrained, a more valid dimensioning criterion is the following. The heat flux (q'') dissipated by the finned conduit can be compared with that (q''_r) removed through a zero thickness flat wall of a reference conduit with the same hydraulic resistance per unit of conduit length and width under the same conditions. As an evaluation parameter, the compared effectiveness of the finned conduit can be defined as follows:

$$E_c = \frac{q''}{q''_r} = \frac{h}{h_r}, \tag{21}$$

h_r being the global heat transfer coefficient of the reference flat wall conduit with the same hydraulic resistance. Since q''_r is uniformly imposed on a wall whose thickness is ideally zero, h_r is coincident with the coefficient of the convective heat transfer between the wall and the fluid. Moreover, since the hydraulic resistance per unit of surface of such a reference conduit is equal to $12\mu/d^3$ and its Nusselt number is equal to 5.385 (Shah and London, 1974), coefficient h_r can be calculated as

$$h_r = 1.176\, k_c \left[\frac{(-dp/dx)e}{\mu\, w_t}\right]^{1/3}. \tag{22}$$

In many heat transfer optimization problems, the dissipator is required to have as small weight or volume as possible. It is then interesting to determine the dimensions and the fin shape which allow Nu_e or E_c to be maximized for a given volume of the finned plate.

4. Finned plate optimization

The geometry of the finned plate is described by parameters a, b, d, e and the profile function $f(y)$. Normalizing all geometrical parameters by d, we get the following dimensionless variables:

$$\alpha = \frac{a}{d},\quad \beta = \frac{b}{d},\quad \epsilon = \frac{e}{d},\quad \phi(\eta) = \frac{f(y)}{d},\quad \eta = \frac{y}{d}. \tag{23}$$

Let us assign a polynomial form to the profile function ϕ:

$$\phi(\eta) = \sum_{i=0}^{n} \psi_i \eta^i. \tag{24}$$

Such a function is univocally determined by the values which it assumes in correspondence with $n+1$ values of η. The values of ϕ in $n+1$ equidistant points on the η axis can then be chosen as the fin profile describing parameters:

$$\phi_i = \phi\left(\frac{i}{n}\alpha\right) \quad \forall i = 0, 1, \ldots, n. \tag{25}$$

Changes in ϕ_i induces in $\phi(\eta)$ variations of a more comparable entity than do changes in ψ_i. For this reason, the first ones are preferable as the fin profile describing parameters instead of the latter ones. Moreover, the average half-width of the fin $\overline{\phi}$ and the average thickness of the finned plate $\overline{\sigma}$ result:

$$\overline{\phi} = \sum_{i=0}^{n} \frac{\psi_i(\phi_0, \ldots, \phi_n)}{i+1} \alpha^i, \tag{26}$$

$$\overline{\sigma} = \beta + \frac{\alpha\, \overline{\phi}}{\epsilon}. \tag{27}$$

Parameter $\overline{\sigma}$ is representative of the volume and the weight of the finned plate.

The heat transfer optimization problem consists now in finding the combination of parameters α, β, ϵ and ϕ_i which allow the maximum Nu_e or E_c to be obtained. To this aim, the following genetic algorithm (Queipo et al., 1994; Fabbri, 1997) can be successfully used.

At the beginning of the algorithm, a population of parameter combinations is generated from a prototype combination. The parameters of the prototype are copied with random mutations. For each combination of the initial population, Nu_e or E_c is then calculated as an evaluation. Afterwards, an established percentage of the combinations is selected on the basis of the best evaluation. The parameters of the selected combinations are then reproduced with random mutation in order to generate a new population which is as numerous as the initial one. The combinations of the new population are valued and selected in the same manner as those of the parental population. The process is iterated until the evaluations no longer improve.

If, after reproduction, some parameter is found to be negative or too small in regard to the structural integrity of the finned plate, it can be resized to the smallest acceptable value. Parameters ϕ_i must be resized together by considering the minimum value ϕ_{min} which $\phi(\eta)$ assumes for η between 0 and α:

$$\widehat{\phi}_i = \phi_{max} - \frac{\phi_{max} - \phi_a}{\phi_{max} - \phi_{min}} (\phi_{max} - \phi_i) \quad \forall i = 0, 1, \ldots, n \tag{28}$$

$\widehat{\phi}_i$ being the new parameter values, ϕ_{max} the maximum assumed by $\phi(\eta)$ and ϕ_a the minimum half-width of the fin which is acceptable.

If the optimization problem consists in finding the parameter combination which presents the highest Nu_e or E_c keeping the finned plate average thickness at an established value $\overline{\sigma}_0$, parameters α, ϵ and ϕ_i can be reproduced with random mutations, while β must be calculated as follows:

$$\beta = \overline{\sigma}_0 - \frac{\alpha\overline{\phi}}{\epsilon}. \tag{29}$$

In this way, β can result as being negative or too small. If that occurs, ϕ_i can be resized or the parameter combination can be rejected by assigning to it a null evaluation.

5. Results

The genetic algorithm described in the previous section has been employed for different optimizations of the finned plate geometry. In every case, populations of 20 samples and a selection percentage equal to 20 were established. During parameter reproduction random errors were introduced, which were uniformly distributed between −10% and +10%. The genetic algorithm was stopped after 40 generations from that time in which an improvement was no longer observed. As a prototype a rectangular fin geometry with β equal to 1/6, ϵ equal to 1/3 and all ϕ_i equal to 1/6 was employed. Diverging and converging trapezoid fin prototypes have also been tested for the third and the fourth polynomial order profile optimization. However, no significant change has been observed in the results, as a consequence of the adoption of such prototypes. For the finite element model, a grid of 16×52 elements (17×53 nodes) was employed. More closed grids have been tested without finding any significant variation in Nu_e or in E_c. In the testing cases, a grid of 20×52 elements produced alterations in Nu_e and E_c of less than 0.1% and 0.2%, respectively, and a grid of 16×65 elements caused changes of less than 0.05% and 0.1%, respectively. In the limit case of α equal to 0, the velocity and the temperature distributions obtainable with the model with the selected grid are in good agreement with the analytical one-dimensional solution and the numerical error on Nu_e and E_c is equal to 0.21%.

Firstly, aiming to maximize the equivalent Nusselt number, some preliminary trials have been carried out by choosing n equal to 0 and leaving all the other parameters free to change. As it was intuitively expected, the algorithm tried to extend the normalized fin height α as much as possible in order to create separated narrow rectangular channels. Aiming to allow a more uniform distribution of the coolant in the conduit, in the subsequent optimizations, α has been constrained to be less than established values. As a consequence of such a limitation, at the end of the algorithm, α has always resulted as being equal to the constraint value.

In Fig. 2, some finned plate geometries obtained with the genetic algorithm by maximizing Nu_e are shown for n ranging from 0 to 4. Parameter γ has been chosen equal to 300 and α constrained to 0.5 and 0.75. The value chosen for γ corresponds to the case of a finned plate made of copper and cooled by water. The describing parameters of the optimum geometries are reported in Table 1 together with the equivalent Nusselt number, the compared effectiveness, the finned plate average thickness and the normalized hydraulic resistance:

$$\zeta = \frac{(-dp/dx)}{(w_t/e)} \bigg/ \frac{12\mu}{d^3} \tag{30}$$

which indicates how many times the hydraulic resistance per unit of length and width of the conduit increases due to the presence of the fins. In Table 1, as well as in Table 2, parameters on the left of the vertical line have been imposed, while those on the right have been found by the genetic optimization algorithm.

The equivalent Nusselt number of the ten geometries of Fig. 2 varies between 14.44 and 87.82 (see Fig. 2 or Table 1). The power removed by the optimized dissipator systems is from 2.7 (= 14.44/5.385) up to 16.3 (= 87.82/5.385) times the heat flux which can be transferred through a flat surface (Nu = 5.385; Shah and London, 1974) at the same distance from the insulated conduit wall such as the fin base. Such improvements depend on the extension of the heat transfer surface and mainly on the alteration of the flow induced by the fins. Considerable differences can be observed between the Nusselt number of the optimum geometry of a polynomial order and that of the optimum geometry whose order is one unit less. The biggest relative differences are between the Nusselt number of the optimum first order and zero order geometry, and between the Nusselt number of the optimum second order and first order geometry. This is true both for α equal to 0.5 and 0.75.

The optimum geometries are based on a compromise between two exigences. The first consists in having, in the cavities between the fins, velocities which are comparable with those at the fin tip or higher. High velocities near the fin tip would in fact cause high thermal gradients in this region, which would lower the bulk temperature without enhancing the heat transfer from the plate base and the lateral surface of the fin. The second exigence consists in maintaining the maximum velocity as close to the dissipator surface as possible, in order to relatively increase the thermal gradient. As a consequence of the first exigence, fins cannot be too closely spaced, for the second one they cannot be too sparse.

Due to this compromise, the optimum fins with α equal to 0.5 are spaced far apart compared to those with α equal to 0.75. Moreover, in both cases, the fin spacing 2ϵ changes with the polynomial order in a nontrivial way due to the complex alterations in the velocity distribution. The plate base thickness β changes consecutively.

The velocity and temperature distributions for the optimum finned plate geometries obtained by maximizing Nu_e are shown in Figs. 3 and 4 for n equal to 0, 2 and 4 and α equal to 0.75. It is evident that the higher order optimum profiles force the maximum velocity to occur in the space between the fins and induce higher thermal gradients near the finned plate base and the fin lateral surface. The density of isothermal lines indicates that the local convective heat transfer coefficient changes along the fin surface and depends on the fin profile.

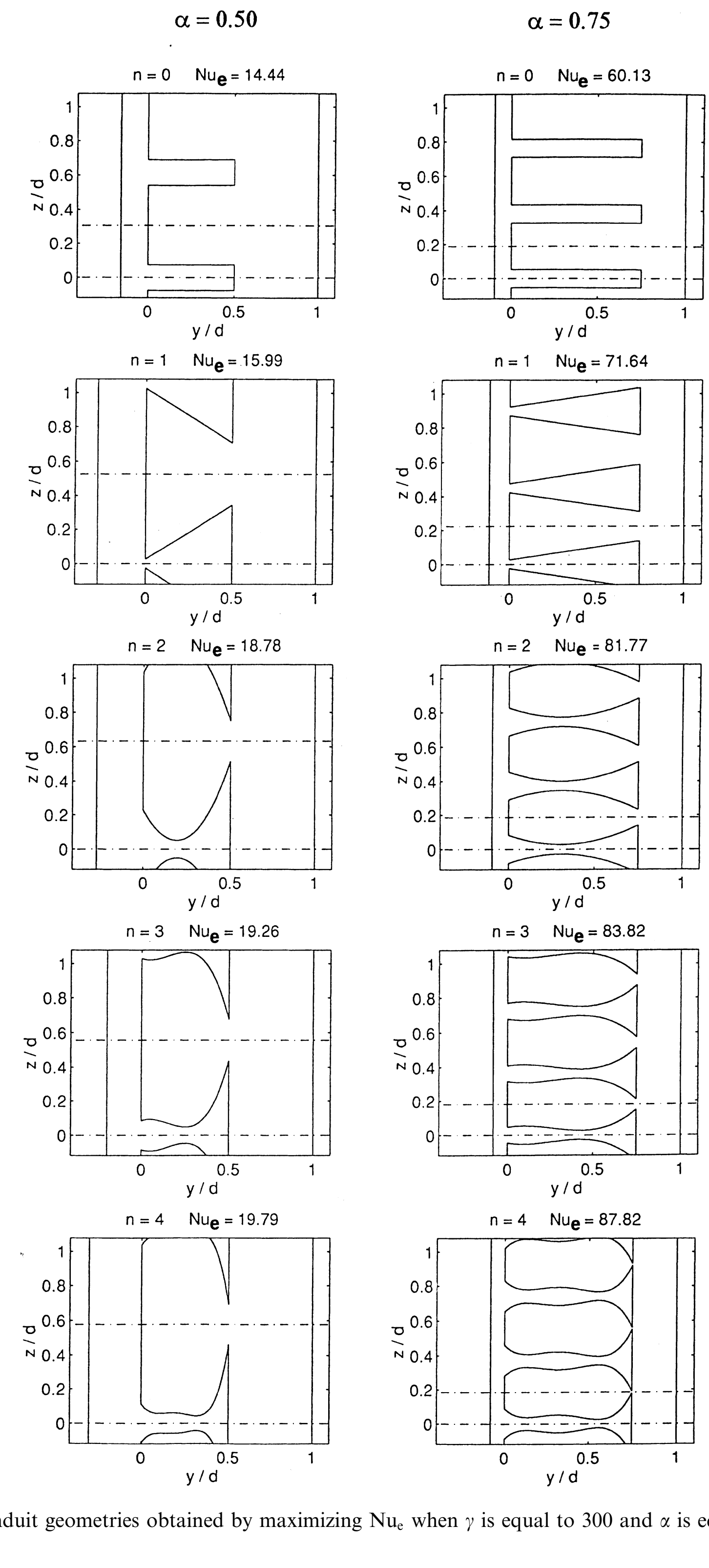

Fig. 2. Finned conduit geometries obtained by maximizing Nu_e when γ is equal to 300 and α is equal to 0.5 and 0.75.

This demonstrates that a mathematical model which considers a constant convective heat transfer coefficient cannot be correctly employed to find the optimum fin profile under the conditions of the present analysis.

In order to compare the performances of the optimum fin profiles of Fig. 2 to those of classical profiles, decreasing triangular and parabolic optimum profiles (Snider and Kraus, 1987) have been investigated under the same conditions. These profiles have been reproduced by letting n be equal to 1 or 2 and by constraining the first derivative of $\phi(\eta)$, in the genetic algorithm, to be negative. For parabolic profiles, the second derivative of $\phi(\eta)$ has also been constrained to be positive. As a result of these constraints, the optimization algorithm gave fin profiles which approximate the rectangular ones of Fig. 2

Table 1
Characteristic parameters of the finned plate geometries obtained by maximizing Nu_e

γ	α	n	$\bar{\sigma}$	β	ϵ	ϕ_0	ϕ_1	ϕ_2	ϕ_3	ϕ_4	Nu_e	E_c	ζ	(Fig.)
300	0.5	0	0.274	0.153	0.307	0.074	–	–	–	–	14.44	1.691	3.984	(2)
300	0.5	1	0.448	0.272	0.525	0.026	0.344	–	–	–	15.99	1.739	4.976	(2)
300	0.5	2	0.397	0.264	0.633	0.231	0.067	0.516	–	–	18.78	2.021	5.139	(2)
300	0.5	3	0.301	0.195	0.556	0.083	0.063	0.078	0.436	–	19.26	2.103	4.922	(2)
300	0.5	4	0.377	0.294	0.576	0.115	0.058	0.052	0.066	0.462	19.79	2.153	4.977	(2)
300	0.75	0	0.305	0.097	0.19	0.053	–	–	–	–	60.13	4.406	16.29	(2)
300	0.75	1	0.391	0.118	0.224	0.026	0.138	–	–	–	71.64	4.857	20.55	(2)
300	0.75	2	0.325	0.095	0.186	0.082	0.03	0.14	–	–	81.77	5.538	20.61	(2)
300	0.75	3	0.296	0.083	0.181	0.045	0.041	0.031	0.151	–	83.82	5.642	21	(2)
300	0.75	4	0.288	0.083	0.185	0.089	0.041	0.049	0.025	0.18	87.82	5.937	20.72	(2)
30	0.75	2	0.45	0.067	0.263	0.187	0.093	0.245	–	–	35.87	2.276	25.08	(5)
30	0.75	4	0.439	0.067	0.267	0.218	0.106	0.117	0.118	0.258	36.45	2.32	24.84	(5)
300	0.75	4	0.2	0.013	0.175	0.073	0.032	0.041	0.024	0.149	83.8	5.677	20.59	(6)
300	0.75	4	0.1	0.006	0.194	0.055	0.017	0.021	0.011	0.102	58.44	4.693	12.37	(6)

Table 2
Characteristic parameters of the finned plate geometries obtained by maximizing E_c

γ	α	n	$\bar{\sigma}$	β	ϵ	ϕ_0	ϕ_1	ϕ_2	ϕ_3	ϕ_4	Nu_e	E_c	ζ	(Fig.)
300	0.5	0	0.235	0.159	0.345	0.052	–	–	–	–	13.86	1.76	3.133	(–)
300	0.5	1	0.314	0.206	0.419	0.021	0.159	–	–	–	15.14	1.826	3.646	(–)
300	0.5	2	0.34	0.229	0.545	0.181	0.045	0.365	–	–	18.23	2.072	4.357	(–)
300	0.5	3	0.344	0.247	0.577	0.088	0.063	0.069	0.415	–	18.92	2.132	4.476	(–)
300	0.5	4	0.322	0.236	0.559	0.131	0.059	0.053	0.079	0.397	19.02	2.161	4.365	(–)
300	0.75	0	0.263	0.103	0.208	0.044	–	–	–	–	57.09	4.618	12.1	(–)
300	0.75	1	0.345	0.117	0.232	0.027	0.114	–	–	–	68.47	5.072	15.75	(–)
300	0.75	2	0.292	0.09	0.202	0.081	0.03	0.127	–	–	78.85	5.731	16.67	(–)
300	0.75	3	0.281	0.097	0.197	0.048	0.038	0.03	0.137	–	80.62	5.843	16.82	(–)
300	0.75	4	0.259	0.08	0.195	0.099	0.037	0.042	0.023	0.153	85.13	6.114	17.29	(–)
30	0.75	2	0.404	0.086	0.284	0.181	0.082	0.211	–	–	33.48	2.407	17.23	(5)
30	0.75	4	0.381	0.071	0.282	0.225	0.098	0.095	0.094	0.239	34.16	2.443	17.5	(5)
300	0.75	4	0.2	0.014	0.202	0.105	0.035	0.041	0.022	0.149	79.21	5.869	15.75	(6)
300	0.75	4	0.1	0.009	0.208	0.049	0.021	0.024	0.011	0.102	56.79	4.735	11.05	(6)

(i.e, with derivatives of $\phi(\eta)$ which are nearly zero). Moreover, by constraining the fin thickness at the fin tip to a low value ($\phi(\alpha) = 0.01$), thin triangular and parabolic profiles have been found. Performances of the two kinds of profiles are almost identical. Nu_e is about 13.5 when α is equal to 0.50, and about 51 when α is equal to 0.75. Therefore, for fins cooled by a laminar flow under the considered conditions, decreasing triangular and parabolic profiles do not perform better than rectangular profiles. This occurs because decreasing triangular and parabolic profiles force the maximum velocity to be near the insulated wall. As a consequence, the highest thermal gradients in the fluid are near the fin tip, where the heat transfer surface is very small.

The increase in the equivalent Nusselt number of the ten geometries of Fig. 2 is accompanied by an increase in the hydraulic resistance. Parameter ζ varies in fact between 3.98 and 20.72, as found from Table 1. Nevertheless, the compared effectiveness reaches values which are noticeably high. A finned plate with a polynomial profile of the fourth order and α equal to 0.75 presents a compared effectiveness equal to 5.937. It means that for a given hydraulic resistance, a conduit of such a geometry is able to remove, under the same condition, a thermal flux which is nearly six times greater than that which could be dissipated by a flat wall channel. Moreover, it is interesting to observe that ζ does not change monotonically with the polynomial order. On the contrary, E_c (as Nu_e obviously does) increases with the polynomial order.

In Table 2, the characteristic parameters of the optimum geometries obtained with the genetic algorithm by maximizing E_c under the same conditions of Fig. 2 are reported. The lateral profile and the thickness of the fins do not differ very much from those obtained by optimizing the equivalent Nusselt number. When α is equal to 0.75, fins which optimize E_c are more largely spaced than those which optimize Nu_e, while that occurs only in two cases when α is equal to 0.5. The compared effectiveness values obtained by maximizing such a parameter are not much higher than those which have previously been found and the equivalent Nusselt numbers are not much lower. Therefore, the geometries shown in Fig. 2 come close to removing a given heat flux with both the lowest distance between the conduit walls and the lowest hydraulic resistance under the considered conditions.

In Fig. 5, some optimum geometries obtained with the genetic algorithm by maximizing Nu_e and E_c are shown for n equal to 2 and 4. In these cases, γ is considered as 30 and α constrained to 0.75. The characteristic parameters of such geometries are reported in Tables 1 and 2. It is evident that when γ is a magnitude order lower than in the previous cases, the improvements in the heat transfer due to the introduction

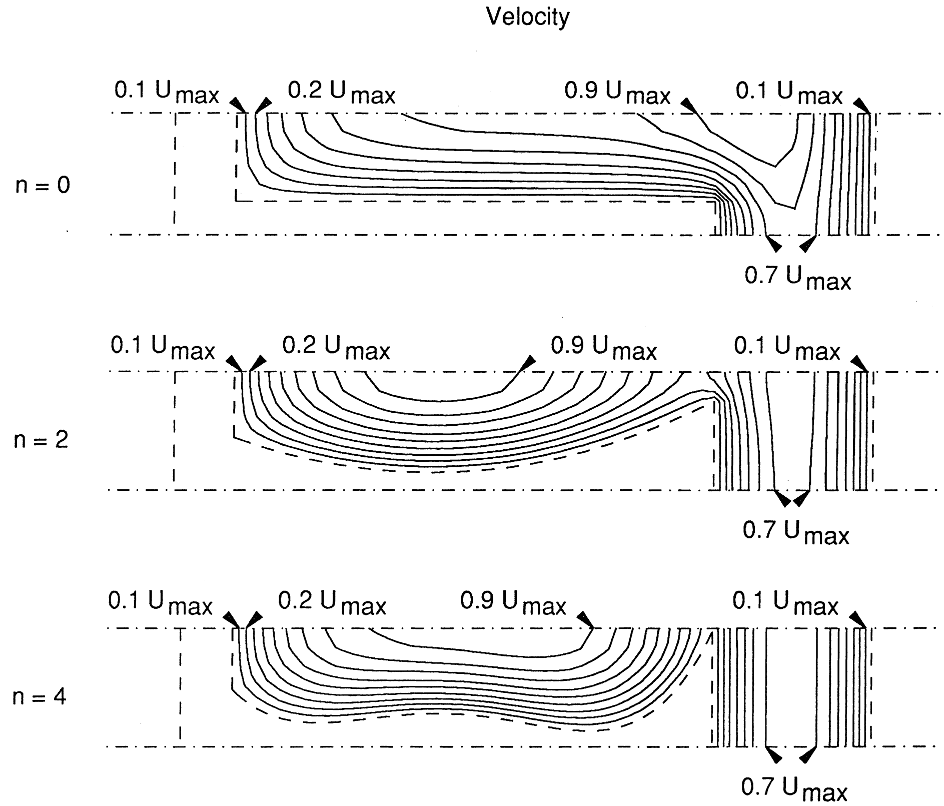

Fig. 3. Velocity distributions in the transversal section of the geometries of Fig. 2 with α equal to 0.75 and n equal 0, 2 and 4. Curves are drawn every 10% of the maximum velocity.

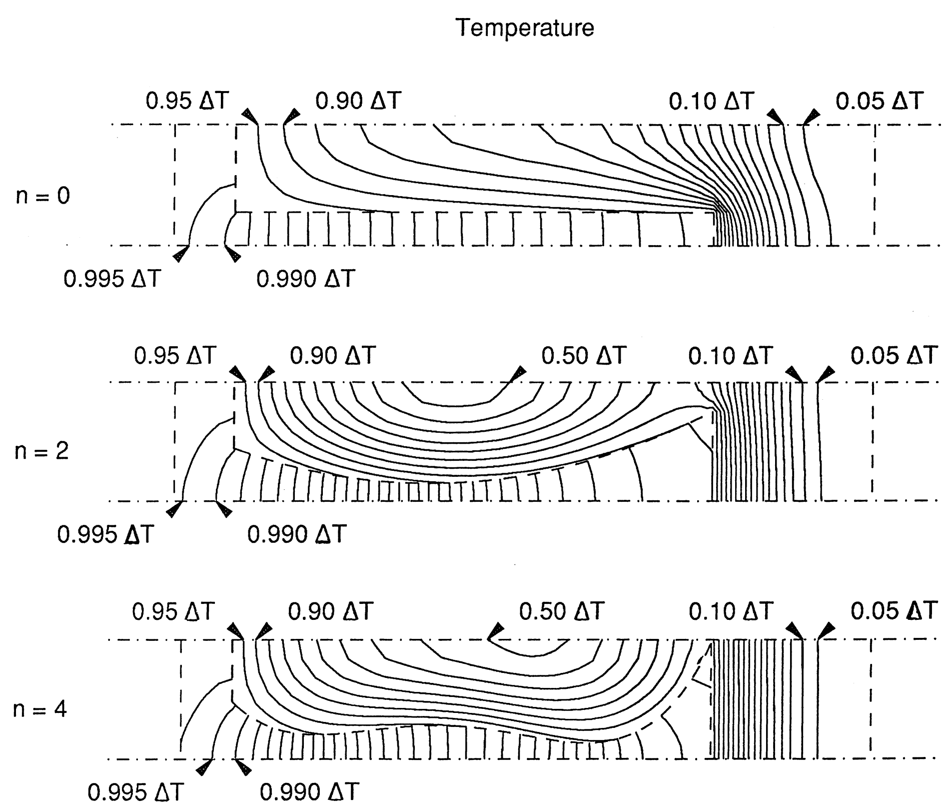

Fig. 4. Temperature distributions in the transversal section of the geometries of Fig. 2 with α equal to 0.75 and n equal 0, 2 and 4. Curves are drawn in the fluid every 5% of the difference between the maximum and minimum temperature and in the solid every 0.5%.

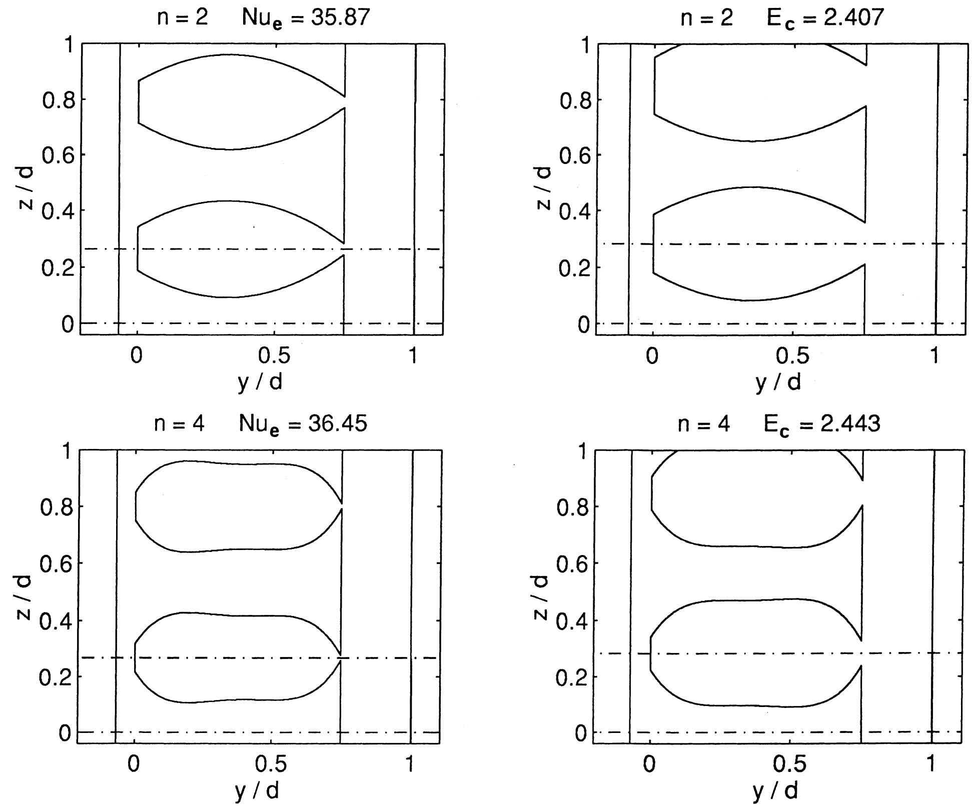

Fig. 5. Finned conduit geometries obtained by maximizing Nu_e and E_c when γ is equal to 30, α is equal to 0.75 and n is equal to 2 and 4.

of the fins are smaller and cannot be significantly enhanced by increasing the polynomial order.

In Fig. 6, the optimum geometries obtained with the genetic algorithm by imposing a constraint value for the finned plate average thickness are shown. In these cases, n is equal to 4, γ is equal to 300 and α is constrained to 0.75. Since the optimum fourth order geometry shown in Fig. 2 for α equal to 0.75 has an average thickness near to 0.28, $\overline{\sigma}$ has been constrained to 0.2 and 0.1. The characteristic parameters of the optimum geometries of Fig. 6 are reported in Tables 1 and 2. When $\overline{\sigma}$ is made to be equal to 0.2, the heat transfer performances of the optimum geometries are not much worse. The geometries which optimize E_c maintain describing parameters which are very similar to those of the unconstrained corresponding one, with the exception of β. When $\overline{\sigma}$ is instead made to be equal to 0.1 the performances become noticeably worse. Parameter β is reduced to very low values.

6. Discussion

The presented optimum geometries demonstrate that it is possible to noticeably increase the heat transfer effectiveness of the plate by assigning a polynomial lateral profile to the fins when the ratio between the thermal conductivity of the solid and the fluid is sufficiently high, as in the case of an aluminium plate cooled by water. A fourth order polynomial profile in optimally spaced fins with a height equal to 0.75 times the channel height provides an increase of 45% in the global heat transfer coefficient referring to the maximum value which can be obtained with a rectangular fin profile (which has been demonstrated as performing better than triangular and parabolic profiles under the considered conditions). Moreover, the heat transfer improvements which are obtainable with a fourth order polynomial profile are not affected that much by little reductions in the finned plate volume, when the thermal conductivity ratio is high. A reduction of 30% from the optimum unconstrained volume causes, in fact, a worsening of less than 5% in the global heat transfer coefficient. On the contrary, when the ratio between the thermal conductivities is a magnitude order lower, the improvements in the heat transfer due to the adoption of a polynomial profile are more limited and a fourth order profile does not perform much better than a second order one. Nevertheless, the presence of the fins in the channel still enhances the heat transfer, also in respect to a flat wall conduit of the same hydraulic resistance.

The temperature distributions which have been shown demonstrate that, under the considered conditions, the local convective heat transfer coefficient considerably changes along the fin surface and depends on the fin profile. Nevertheless, the obtained results partially confirm the predictions of theoretical analyses carried out by assigning a constant value to the local convective heat transfer coefficient. In particular, in the present analysis, the best fin performances have been observed in correspondence with wavy fin profiles. Such a result is in accordance with the predictions of Maday (1974), Snider and Kraus (1987), Snider et al. (1990), Fabbri and Lorenzini (1995), Fabbri (1997). On the other hand, the wavy profiles found in the present analysis are flatter in the middle of the fin than the optimum polynomial fin profiles which we had found with a genetic algorithm by assigning a constant value to the convective heat transfer coefficient (Fabbri, 1997). Moreover, the quasi-linear relationship between the optimum fin effectiveness and the fin profile polynomial order observed in our previous work has not been confirmed under the conditions of the present analysis. By increasing the polynomial order n, in fact, under the conditions considered in this work, the incremental improvements in the heat transfer of the optimum fin

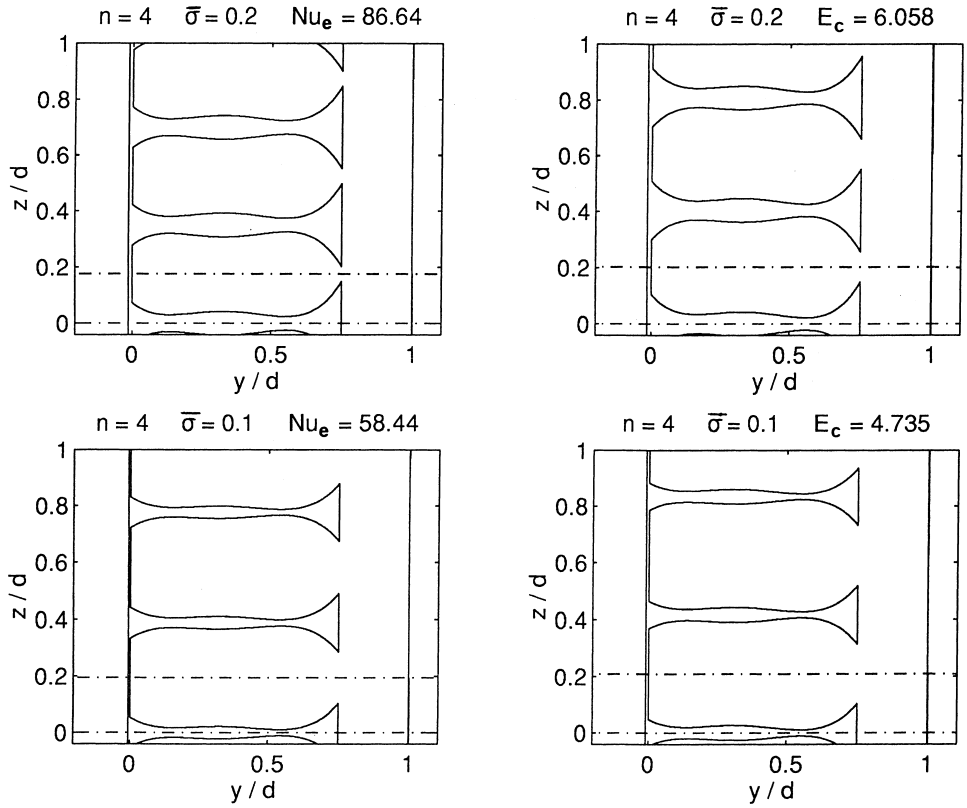

Fig. 6. Finned conduit geometries obtained by maximizing $\mathrm{Nu_e}$ and E_c when γ is equal to 300, α is equal to 0.75, n is equal to 2 and 4 and the dissipator volume is constrained.

profiles are greater when n is low ($n = 0$, $n = 1$) and tend to reduce when n is greater.

The idealizations which have been introduced in the mathematical model utilized in the genetic algorithm limit the practical validity of the present analysis. In particular, the optimum geometries which have been found are convenient for sufficiently long conduits, whose entrance length is short in respect to the total length. Moreover, the idealization of uniform fluid properties can introduce some errors in the solution of the problem of optimizing the fin spacing and shape. This occurs when viscosity and thermal conductivity are strongly sensitive to temperature changes. If viscosity decreases with temperature, in fact, velocity becomes relatively higher near the fins than in the case in which viscosity is constant. As a result, the convective heat transfer coefficient increases and the improvements in the heat transfer which can be obtained by optimizing the fin profile (Fabbri, 1997) are reduced. A similar effect produces the dependence of thermal conductivity on temperature. In the studied cases, temperature varies much more in the fluid than in the solid. Therefore, the dependence of thermal conductivity on temperature in the solid does not noticeably affect the the fin effectiveness (Huang and Shah, 1992). On the contrary, such a dependence in the fluid can influence the optimization of the finned plate geometry. If thermal conductivity in the fluid increases with the temperature, thermal gradients decrease near the fins and increase in the cooler region. As a result, since the transferred conductive heat flux is greater near the fin, the temperature distribution becomes more uniform and less sensitive to alterations in the fin shape.

Therefore, a more rigorous solution to the problem of optimizing the heat transfer through a finned plate cooled by a laminar flow could be obtained by considering temperature dependent parameters. For the case in which longitudinal conduction hardly influences the heat transfer, the proposed mathematical model could still be utilized by solving systems of Eqs. (9) and (17) iteratively. The same optimization algorithm could probably be successfully employed.

7. Conclusions

An optimization algorithm is proposed for fin profile and spacing to maximize with some constraints the heat flux removed through a finned plate cooled in laminar flow. In particular, for given fin relative height and dissipator volume, the algorithm can find the finned plate geometries which maximize the heat flux dissipated with the lowest distance between the conduit walls or with the lowest hydraulic resistance. The mathematical model utilized in the optimization algorithm takes into account changes which are induced by the alteration of the fin spacing and profile in the convective heat flux leaving the fin surface. The heat transfer problem is simultaneously solved in the solid and in the fluid. Therefore, no convective heat transfer coefficient is explicitly considered for the solid surface in contact with the fluid, and no hypothesis on this parameter is directly introduced.

References

Bar-Cohen, A., Kraus, A.D., 1990. Advances in thermal modeling of electronic components and systems, Chapter 3. ASME Press Series, New York.

Chung, B.T.F., Iyer, J.R., 1993. Optimum design of longitudinal rectangular fins and cylindrical spines with variable heat transfer coefficient. Heat Transfer Eng. 14 (1), 31–42.

Duffin, R.J., 1959. A variational problem relating to cooling fins. J. Math. Mech. 8, 47–56.

Fabbri, G., Lorenzini, G., 1995. Analisi numerica bidimensionale di dissipatori a profilo sinusoidale. In: Proceedings of 13th UIT National Congress. Bologna, Italy, pp. 491–499.

Fabbri, G., 1997. A genetic algorithm for fin profile optimization. Int. J. Heat Mass Transfer 40, 2165–2172.

Huang, L.J., Shah, R.K., 1992. Assessment of calculation methods for efficiency of straight fins of rectangular profile. Int. J. Heat Fluid Flow 13, 282–293.

Kays, W.M., London, A.L., 1984. Compact heat exchangers, 3rd ed. Chapter 1, McGraw-Hill, New York.

Yeh, R.H., 1994. Optimum spines with temperature dependent thermal parameters. Int. J. Heat Mass Transfer 37, 1877–1884.

Maday, C.J., 1974. The minimum weight one-dimensional straight fin. ASME J. Eng. Ind. 96, 161–165.

Queipo, N., Devarakonda, R., Humphrey, J.A.C., 1994. Genetic algorithms for thermosciences research: Application to the optimized cooling of electronic components. Int. J. Heat Mass Transfer 37, 893–908.

Schmidt, E., 1926. Die Wärmeübertragung durch Rippen. Ver. Dt. Ing. 70, 885–951.

Shah, R.K., London, A.L., 1974. Thermal boundary conditions for laminar duct flow forced convection. ASME J. Heat Transfer 96, 159–165.

Snider, A.D., Kraus, A.D., 1987. The quest for the optimum longitudinal fin profile. Heat Transfer Eng. 8 (2), 19–25.

Snider, A.D., Kraus, A.D., Graff, S., Rodriguez, M., Kusmierczyk, A.G., 1990. Optimal fin profiles, Classical and modern. In: Proceedings of the Ninth International Conference on Heat Transfer, vol. 4. 1990, pp. 15–19.

Spiga, M., Fabbri, G., 1994. Efficienza di dissipatori a profilo sinusoidale. In: Proceedings of the 12th UIT National Congress. L'Aquila, Italy, pp. 197–204.

Tsukamoto, Y., Seguchi, Y., 1984. Shape optimization problem for minimum volume fin. Heat Transfer Japanese Research 13 (1), 1–19.